- **Definition of *j***
- **Conjugate**
- **Vector Representation**
- **Euler's Formula**

- **"*j*"**

$$j \triangleq +\sqrt{-1}$$

$$j^2 = -1, \qquad j^3 = -j, \qquad j^4 = 1$$

General complex number:

$$z = x + j\,y \qquad x,\, y \in \mathbf{R} \qquad \text{Cartesian form}$$
$$z = r\,e^{j\theta} \qquad r,\, \theta \in \mathbf{R} \qquad \text{Polar form}$$

x **is the** real **part of** z, **and** y **is the** imaginary **part of** z:

$$\mathrm{Re}\{z\} \triangleq x$$
$$\mathrm{Im}\{z\} \triangleq y$$

• Conjugate

replaces all occurences of *j* with *–j*:

$$\overline{x + jy} = x - jy$$
$$\overline{e^{jt}} = e^{-jt}$$

a number times its conjugate is real:

$$z\,\overline{z} = (x + j\,y)(x - j\,y) = x^2 + y^2$$
$$z\,\overline{z} = (r\,e^{j\theta})(r\,e^{-j\theta}) = r^2$$

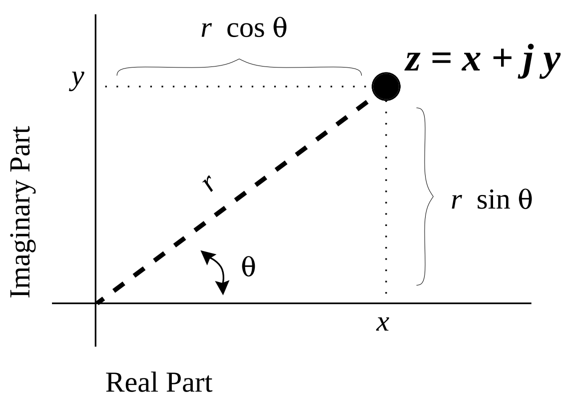
division of complex numbers:

$$\frac{z_1}{z_2} = \frac{z_1}{z_2} \cdot \frac{\overline{z_2}}{\overline{z_2}} = \frac{\text{complex number}}{\text{real number}}$$

complex roots of real polynomials come in conjugate pairs:

$$x^3 + x^2 + x + 1 = (x + 1)\,\underbrace{(x + j)(x - j)}$$

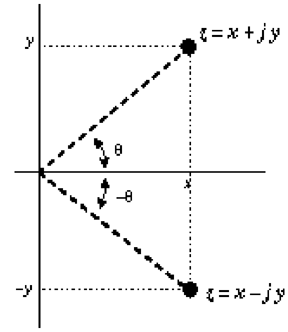
• Complex numbers as two-dimensional vectors



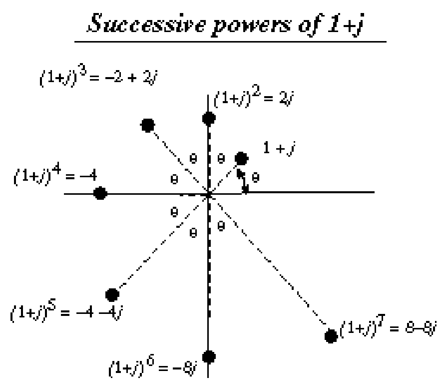
Note the equivalence between representing the complex number in Cartesian and Polar forms:

$$z = x + j\,y = r\,e^{j\theta}$$
$$y = r\sin\theta$$
$$x = r\cos\theta$$
$$x + j\,y = \underbrace{r(\cos\theta + j\,\sin\theta) = r\,e^{j\theta}}$$

– Conjugate of a number is the mirror image on the x-axis.



– Successive powers of a complex number rotate the complex vector around the origin.

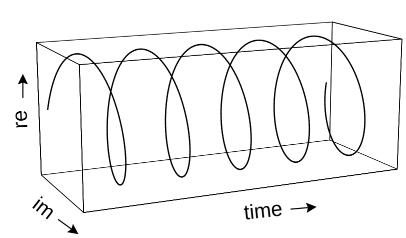


• Complex Exponentials

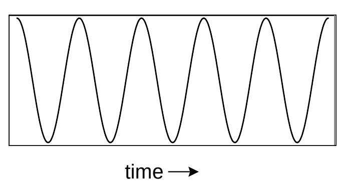
Euler's Formula:

$$e^{jt} = \cos t + j \sin t$$

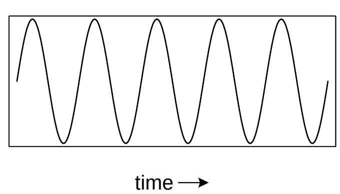
Complex Sinusoid



Real Part (cosine)

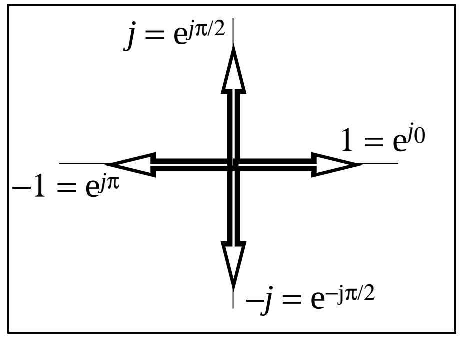


Imaginary Part (sine)



• **Euler's Identity:**

$$\boxed{e^{j\pi} = -1}$$



Proof: Let $\theta = \pi$, then

$$\begin{aligned} e^{j\theta} &= \cos\theta + j\ \sin\theta \\ &= \underbrace{\cos\pi}_{-1} + j\ \underbrace{\sin\pi}_{0} \\ &= -1 \end{aligned}$$

De Moivre's Theorem:

$$(\cos\theta + j\ \sin\theta)^n = \cos n\theta + j\ \sin n\theta$$