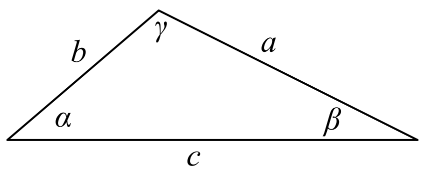
8.1 The Law of Sines

The Law of Sines is basically the ratio of the sine of an angle in degrees and the length of the side opposite that angle equal to the sine of another angle and the length of the side opposite that angle. This formula is used to solve triangles given at least the measure of one angle and side.



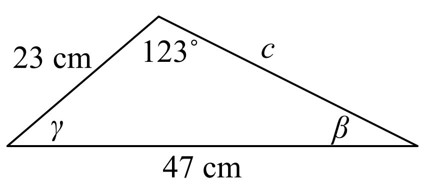
Law of Sines : $\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$

When solving a triangle, make sure your calculator is in degree mode.

SSA Variations Given two sides and its consecutive angle, there are some cases to keep in mind when deciding the solution.

Angle α	a $h = b\sin\alpha$	Number of triangles
Acute	$0 < a < h$	0
Acute	$a = h$	1
Acute	$h < a < b$	2
Acute	$a \geq b$	1
Obtuse	$0 < a \leq b$	0
Obtuse	$a > b$	1

Example Given the triangle below, solve for the missing parts.



Examining the triangle we see that α is obtuse and that $a > b$. Therefore, we are solving for one triangle. Now we can apply the Law of Sines to solve the triangle.

Law of Sines states: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$\frac{\sin 123°}{47} = \frac{\sin\beta}{23} = \frac{\sin\gamma}{c}$ can be separated by keeping the ratio we do know and set it equal to the ratio where part is known.

$$\frac{\sin 123°}{47} = \frac{\sin\beta}{23}$$
$$23 \cdot \frac{\sin 123°}{47} = \sin\beta$$
$$\frac{23\sin 123°}{47} = \sin\beta$$
$$0.4104 = \sin\beta$$

$\beta = \sin^{-1}(0.4104) = 24°$

$B = 24°$

Now that we know two angles of the triangle, we are able to determine the measure of the third angle since the sum of the angles of any triangle is 180˚.

$180° - (24° + 123°) = 180° - 147° = 33°$

$\gamma = 33°$

Solving for the length of side c by applying the Law of Sines.

$$\frac{\sin 123°}{47} = \frac{\sin 33°}{c}$$

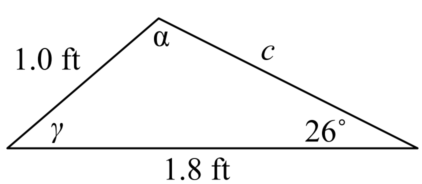
$$c \sin 123° = 47 \sin 33°$$

$$c = \frac{47\sin 33°}{\sin 123°} = 30.5 = 31 \text{ cm}$$

$c = 31$ cm

Example Solve the triangle(s) with $\beta = 26°$, $a = 1.8$ ft, and $b = 1.0$ ft.

First draw and label what you know about the triangle. Then you can solve as we did above.



Notice this time we are given β and not α. Let β represent the angle α.

Examining the triangle we see that α is acute and that $h = 1.8\sin 26^\circ$, which is approximately 0.8. Therefore, we are solving for two triangles because $h < a < b$. Now we can apply the Law of Sines to solve the triangle.

Law of Sines states: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$\frac{\sin 26^\circ}{1.0} = \frac{\sin\alpha}{1.8} = \frac{\sin\gamma}{c}$ can be separated by keeping the ratio we do know and set it equal to the ratio where part is known.

$$\frac{\sin 26^\circ}{1.0} = \frac{\sin\alpha}{1.8}$$

$$1.8 \cdot \frac{\sin 26^\circ}{1.0} = \sin\alpha$$

$$\frac{1.8\sin 26^\circ}{1.0} = \sin\alpha$$

$\sin\alpha = 0.7891$

$\alpha = \sin^{-1}(0.7891) = 52^\circ$ or $\alpha = 180^\circ - 52^\circ = 128^\circ$

$\alpha = 52^\circ$	$\alpha = 128^\circ$

Now that we know two angles of the triangle, we are able to determine the measure of the third angle since the sum of the angles of any triangle is 180˚.

$180^\circ - (52^\circ + 26^\circ) =$
$180^\circ - 78^\circ = 102^\circ$

or

$180^\circ - (128^\circ + 26^\circ) =$
$180^\circ - 154^\circ = 26^\circ$

$\gamma = 102^\circ$	$\gamma = 26^\circ$

Solving for the length of side c on both possible triangles by applying the Law of Sines.

$\frac{\sin 26^\circ}{1.0} = \frac{\sin 102^\circ}{c}$ or $\frac{\sin 26^\circ}{1.0} = \frac{\sin 26^\circ}{c}$

$c\sin 26^\circ = 1.0\sin 102^\circ$ $\qquad$ $c\sin 26^\circ = 1.0\sin 26^\circ$

$c = \frac{1.0\sin 102^\circ}{\sin 26^\circ} = 2.2$ ft $\qquad$ $c = \frac{1.0\sin 26^\circ}{\sin 26^\circ} = 1.0$ ft

$c = 2.2$ ft	$c = 1.0$ ft

Try the following:

1. Solve the triangle(s) with $\beta = 98^\circ$, $a = 62$ meters, and $b = 88$ meters.

2. Solve the triangle(s) with $a = 8$ km, $b = 10$ km, and $\alpha = 35^\circ$.

3. Solve the triangle(s) with $\alpha = 123.2^\circ$, $a = 101$ yd, and $b = 152$ yd.

Answers: $\alpha = 44^\circ$, $\gamma = 38^\circ$, and $c = 55$ m ; $\beta = 134^\circ$, $\gamma = 11^\circ$, and $c = 2.7$ km **or** $\beta = 46^\circ$, $\gamma = 99^\circ$, and $c = 14$ km ; no triangle is possible