Voltage buildup in a DC shunt generator

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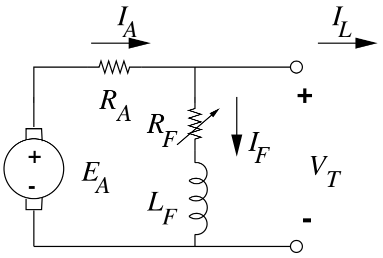
1 No-load terminal voltage buildup

Shunt generators have a distinct advantage over separately excited dc generators in that no external power supply is required for the field circuit.



The voltage buildup in a dc shunt generator depends on the presence of a *residual flux* in the poles of the generator. When the generator first starts to turn, an internal voltage will be generated which is given by

$$E_A = K\phi_{\text{res}}\omega.$$

This voltage appears at the terminals of the generator. But when this voltage appears at the terminals, it causes a current to flow in the generator's field coil. Note that there is no load connected to the terminals, hence the field current I_F is only current caused by the voltage E_A. This field current produces a magnetomotive force in the poles, which increases the flux in them. The increase in flux causes an increase in E_A and so on.

This process can be modeled mathematically using a differential equation. Since the internal voltage E_A, the flux ϕ in the machine and the field current I_F change while the voltage is building up, these quantities should be treated as time-varying variables:

$$E_A = E_A(t), \quad \phi = \phi(t), \quad I_F = I_F(t).$$

At no-load, the transient behavior of a shunt generator is described by the equations:

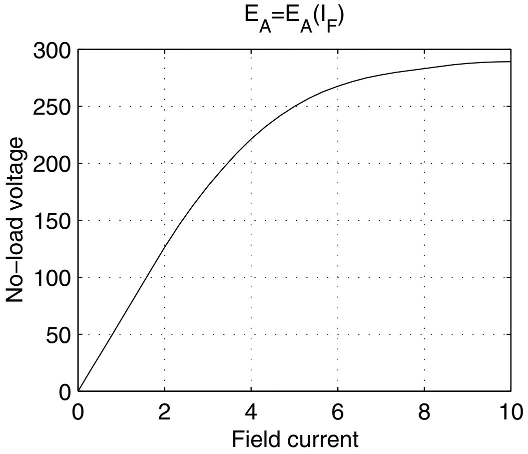
$$\begin{aligned} E_A &= V_T + R_A I_A \\ I_A &= I_F + I_L = I_F \\ V_T &= R_F I_F + N_F \frac{d\phi}{dt} \\ &= R_F I_F + \frac{N_F}{K\omega} \cdot \frac{dE_A}{dt}; \end{aligned} \tag{1}$$

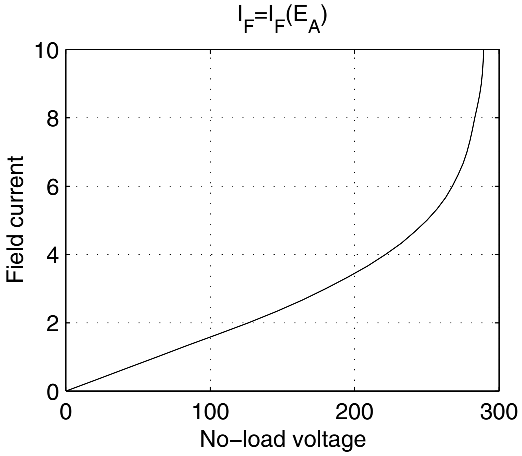
see the schematics above. Here, N_F is the number of turns in the field coil. In the last equation, we take into account the fact that during the voltage buildup, the flux ϕ in the machine is time-changing. Hence, this flux induces a voltage across the inductor L_F. This voltage must be included in the KVL equation.

After some algebra, equations (1) can be reduced to the following differential equation:

$$\frac{N_F}{K\omega} \cdot \frac{dE_A}{dt} \quad = \quad E_A - (R_A + R_F) I_F(E_A), \tag{2}$$

Here $I_F(E_A)$ is the function which expresses the field current versus the internal generated voltage E_A. This function is an inverse function to the function $E_A = E_A(I_F)$ which is based on the dc magnetization curve; see the figure given below.





The initial condition for the differential equation (2) is given by the initial internal voltage value

$$E_A(0) = K\phi_{\text{res}}\omega. \tag{3}$$

The nonlinear differential equation (2) can be analyzed using the linearization technique[1]

The steady-state solution of the differential equation corresponds to $\frac{E_A}{dt} = 0$. This is the *point of equilibrium* where the right-hand side of the differential equation equals to zero:

$$E_A = (R_A + R_F) I_F(E_A), \quad \text{or} \quad I_F(E_A) = \frac{E_A}{R_A + R_F}.$$

Graphically, the solution to the above equation can be found as the point where the straight line $I_F = \frac{E_A}{R_A + R_F}$ intersects the curve $I_F = I_F(E_A)$; see the graph given below.

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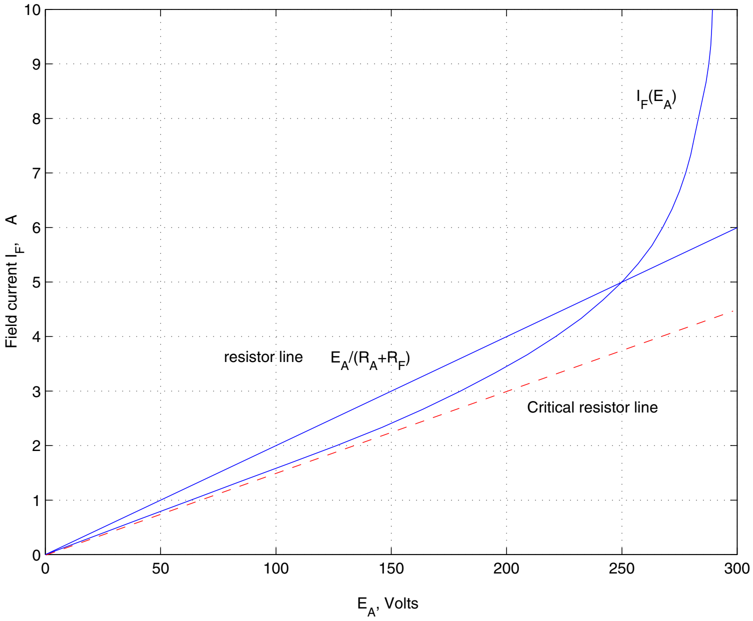
[1] Linearization is a technique which allows one to study stability of steady-state solutions of differential equations. This technique is introduced and explained in the *Control Theory* course. Students are assumed to be familiar with this technique.



From the above figure, one can notice that there is a critical value of the field resistance at which the slope of the curve $I_F(E_A)$ at the origin and the slope of the line $I_F = \frac{E_A}{R_A+R_F}$ are equal. Note that the slope of the curve $I_F(E_A)$ at the origin is given by $I'_F(0)$. Hence, the critical field resistance is determined by the equation

$$\frac{1}{R_A + R_F} = I'_F(0) \quad \text{i.e.,} \quad R_{F,\text{critical}} = \frac{1}{I'_F(0)} - R_A.$$

Furthermore, if $R_F < R_{F,\text{critical}}$, and therefore the slope of the straight line is greater than the slope of the curve $I_F(E_A)$, $\frac{1}{R_A+R_F} > I'_F(0)$, then there are two solutions to the equation $E_A = (R_A + R_F)I_F(E_A)$:

$$E_A = 0 \quad \text{and} \quad E_A = E_A^* \neq 0;$$

the value of E_A^* can be found numerically if necessary. These points are the equilibrium points of the differential equation (2).

Alternatively, if $R_F \geq R_{F,\text{critical}}$ and $\frac{1}{R_A+R_F} < I'_F(0)$, then the straight line goes all the way below the curve $I_F(E_A)$. In this case the curve $I_F(E_A)$ is intercepted by the line $I_F = \frac{E_A}{R_A+R_F}$ at the origin only. Hence, in this case the differential equation has a unique equilibrium:

$$E_A = 0.$$

We now investigate the behavior of the solution of the differential equation (2) with initial condition (3) about each equilibrium using the linearization technique. In what follows, $\tilde{E}_A$ will denote the variable of the linearized equation. Usually, this variable is defined as $\tilde{E}_A = E_A - E_{A,\text{equilibrium}}$. Hence, the linearized equation will always have the equilibrium located at the origin.

Case $R_F < R_{F,\text{critical}}$. *Linearization about* $E_A = 0$. Let $\tilde{E}_A = E_A$. The linearized equation is

$$\frac{N_F}{K\omega} \cdot \frac{d\tilde{E}_A}{dt} = \tilde{E}_A(1 - (R_A + R_F)I'_F(0)).$$

Since $R_F < R_{F,\text{critical}}$, then at $E_A = 0$, the straight line $\frac{1}{R_F+R_A}$ has a slope steeper than that of the curve $I_F(E_A)$. That is, $\frac{1}{R_F+R_A} > I'_F(0)$. Thus, for any initial internal voltage $\tilde{E}_A = E_A > 0$ $\frac{d\tilde{E}_A}{dt} > 0$ and hence, the equilibrium $E_A = 0$ of the underlying differential equation is unstable.

Linearization about $E_A = E_A^*$. Let $\tilde{E}_A = E_A - E_A^*$. The linearized equation is

$$\frac{N_F}{K\omega} \cdot \frac{d\tilde{E}_A}{dt} = \tilde{E}_A(1 - (R_A + R_F)I'_F(E_A^*)).$$

The slope of the curve $I_F(E_A)$ at $E_A = E_A^*$ is steeper than that of the straight line:

$$I'_F(E_A^*) > \frac{1}{R_F + R_A}.$$

Thus, $\frac{d\tilde{E}_A}{dt} < 0$ if $\tilde{E}_A > 0$, i.e., $E_A > E_A^*$ and also, $\frac{d\tilde{E}_A}{dt} > 0$ if $\tilde{E}_A < 0$ i.e., $E_A < E_A^*$. Hence, the equilibrium $E_A = E_A^*$ is stable.

Since the initial voltage $E_A(0) = K\phi_{\text{residual}}\omega$ is very low, it can be assumed that $E_A(0) < E_A^*$. Therefore, E_A will increase as $t \to \infty$ until $E_A \approx E_A^*$.

Case $\boldsymbol{R_F \geq R_{F,\text{critical}}}$. In this case, we linearize about $E_A = 0$ only as this is a unique equilibrium. Let $\tilde{E}_A = E_A$. The linearized equation is

$$\frac{N_F}{K\omega} \cdot \frac{d\tilde{E}_A}{dt} = \tilde{E}_A(1 - (R_A + R_F)I'_F(0)).$$

Since $R_F > R_{F,\text{critical}}$, then the slope of the curve $I_F(E_A)$ at $E_A = 0$ goes all the way above the straight line. In this case, for any $\tilde{E}_A > 0$, we have $\frac{d\tilde{E}_A}{dt} < 0$. That is, the voltage E_A decays to zero. Hence, the equilibrium $E_A = 0$ is stable. Voltage does not build up in this case.

2 Terminal characteristic of a DC shunt generator

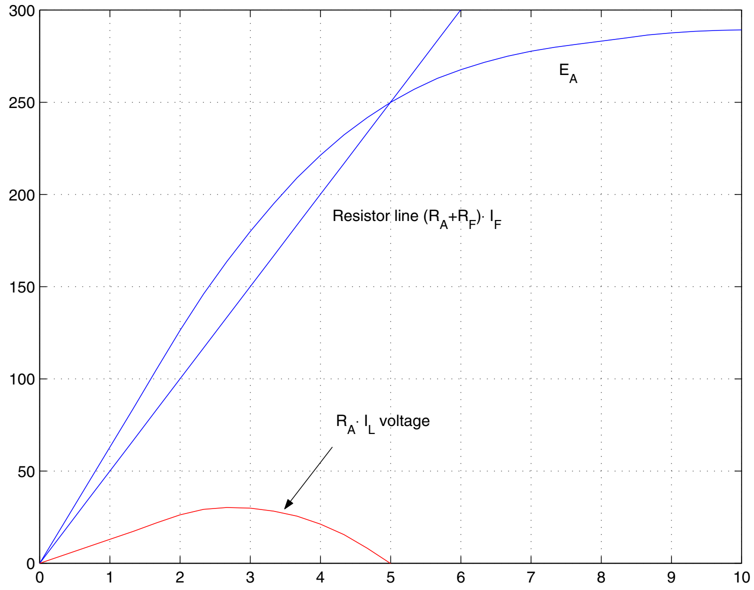
The steady-state operation of a loaded generator is described by the equations:

$$\begin{aligned} E_A &= V_T + I_A R_A \\ V_T &= R_F I_F \\ I_A &= I_L + I_F. \end{aligned}$$

From these equations,

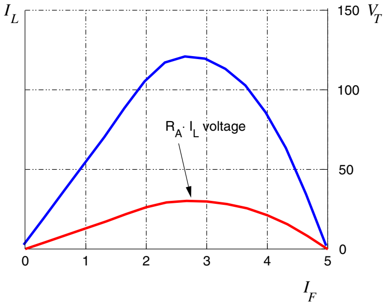
$$R_A I_L = E_A - (R_A + R_F)I_F$$

That is, the $R_A I_L$ is the voltage gap between the no load voltage and the voltage drop across R_A and R_F caused by the field current I_F; see the graph below.

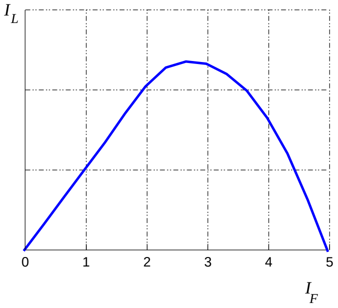


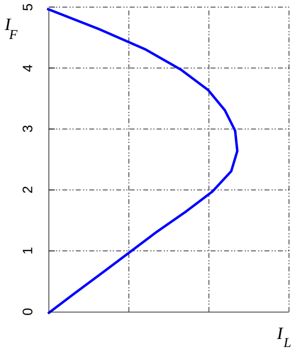
Using this graph, the graph V_T versus I_L can be obtained using the following transformations:

1. Produce the graph I_L versus I_F by dividing the previous graph by R_A;

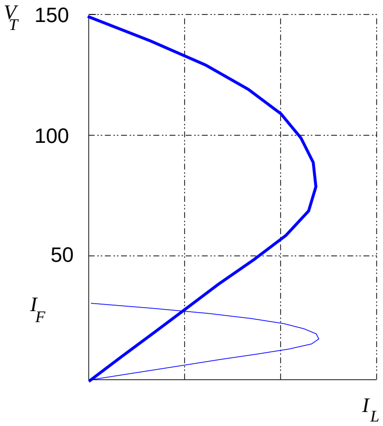


2. Produce the graph I_F versus I_L by swapping the coordinate axes (mirroring the graph about $45°$ axis).





3. Produce the graph $V_T = I_F R_F$ versus I_L by multiplying the previous graph by R_F;


V_T
150
100
50
I_F
I_L